

## Entropy function and $AdS_2/CFT_1$ correspondence

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**ABSTRACT:** Wald's formula for black hole entropy, applied to extremal black holes, leads to the entropy function formalism. We manipulate the entropy computed this way to express it as the logarithm of the ground state degeneracy of a dual quantum mechanical system. This provides a natural definition of the extremal black hole entropy in the full quantum theory. Our analysis also clarifies the relationship between the entropy function formalism and the Euclidean action formalism.

**KEYWORDS:** Black Holes in String Theory, AdS-CFT Correspondence.

Wald's formula [1–4] gives an expression for the entropy of a black hole in terms of the field configurations near the horizon in any general coordinate invariant theory of gravity including those with higher derivative terms in the action. For an extremal black hole whose near horizon geometry has an  $AdS_2$  factor, this formula may be encoded in the entropy function formalism that reduces the problem of computing the entropy into a purely algebraic problem for spherically symmetric black holes [5] and a problem involving solution of simple differential equations for rotating black holes [6] (see [7] for a review and other references).

The original Wald formula was derived from classical considerations. For various reasons one would like to find a generalization of this formula in the quantum theory. In fact once we begin including higher derivative corrections to the action in string theory, the notion of classical action becomes ambiguous since in different equivalent descriptions of the theory related by duality the classical actions differ. Thus black hole entropy computed using the classical action does not respect the duality symmetries of the theory. Various approaches to addressing this problem have been suggested, e.g. using the one particle irreducible (1PI) effective action instead of the classical action in computing the entropy [8–11, 7], or, in the special case of  $\mathcal{N} \geq 2$  supersymmetric string theories in four dimensions, the OSV formula [12–15].

We shall follow a different approach to this problem. Our goal will be to begin with the expression for the entropy computed in the classical theory, and relate it via  $AdS/CFT$  correspondence [16–18] to the logarithm of the ground state degeneracy in a dual quantum mechanics living on the boundary of  $AdS_2$ . The latter can then be regarded as the definition of the extremal black hole entropy in the full quantum theory. Related discussion on  $AdS_2/CFT_1$  correspondence can be found in [19–28].

Before describing the details of our analysis, it will be prudent to summarize its physical content. The original Wald's formula for black hole entropy holds for non-extremal black holes, and in applying this result to extremal black holes we must define the entropy of an extremal black hole as a limit of the entropy of a non-extremal black hole. In particular the entropy function formalism computes the entropy of an extremal black hole only in this sense. This can be given an alternate interpretation as follows. If we consider a black hole that is close to being extremal, we expect its near horizon geometry to develop a long  $AdS_2$  throat, but unlike in the case of an extremal black hole where the throat is infinitely long, the throat for a non-extremal black hole will be capped by a regular horizon. Nevertheless since the throat can be made as long as we want by going close to extremality, we expect that we should be able to decouple the asymptotic region from the  $AdS_2$  throat and view the near horizon configuration as a black hole solution in  $AdS_2$  [21, 22, 29, 30, 26]. This solution, known as the Jackiw-Teitelboim black hole [31–33], has been displayed explicitly in eq. (9). The entropy computed by the entropy function method can be interpreted as the entropy of this black hole solution in  $AdS_2$ . We find that this agrees with the entropy computed using the Euclidean action approach [34], in agreement with the general result that for a regular black hole solution the Wald entropy and the one computed using the Euclidean action formalism coincide [1, 35]. The Euclidean action on the other hand can be related to the partition function of the quantum mechanics living on the boundary of  $AdS_2$

via the *AdS/CFT* correspondence. By studying in detail the temperature dependence of the bulk and the boundary side of the computation one finds that the Wald entropy can be interpreted as the logarithm of the ground state degeneracy of this dual quantum mechanics. The latter can then be taken as the definition of the entropy of an extremal black hole even in the full quantum theory.

We begin with a brief review of the entropy function formalism in a classical theory of gravity.<sup>1</sup> Extremal black holes will be defined to be those with an *AdS*<sub>2</sub> factor in the near horizon geometry [36, 37]. More precisely the near horizon geometry of an extremal black hole will have the structure of a compact space *K*, containing the compact directions of the theory and also the angular coordinates of space-time, fibered over an *AdS*<sub>2</sub> space labelled by the time coordinate *t* and the radial variable *r*. It will be useful to regard the theory in this background as a two dimensional theory obtained as a result of compactifying the fundamental theory on *K* [6]. We can then describe the dynamics in the near horizon geometry of the black hole by a theory of gravity coupled to a set of abelian gauge fields  $A_\mu^{(I)}$  and a set of neutral scalar fields  $\{\phi_s\}$ , integrating out all other fields. Let  $\mathcal{L}_0$  be the classical Lagrangian density and  $\Gamma_0$  be the classical action describing the dynamics of these massless fields:

$$\Gamma_0[g_{\mu\nu}, \{A_\mu^{(I)}\}, \{\phi_s\}] = \int d^2x \sqrt{-\det g} \mathcal{L}_0. \quad (1)$$

In this theory we consider a general field configuration consistent with the SO(2, 1) isometry of *AdS*<sub>2</sub>. This is of the form:

$$ds^2 = v \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right), \quad \phi_s = u_s, \quad F_{rt}^{(I)} = e^I, \quad (2)$$

where  $F_{\mu\nu}^{(I)} = \partial_\mu A_\nu^{(I)} - \partial_\nu A_\mu^{(I)}$  and *v*,  $\{u_s\}$  and  $\{e^I\}$  are constants labelling the background. Note that there are no parameters explicitly labelling the magnetic charges; they are encoded in the components of the gauge field strengths along the compact directions and appear as discrete parameters labelling the two dimensional theory. We now define:

$$f(\vec{u}, v, \vec{e}) \equiv \sqrt{-\det g} \mathcal{L}_0 = v \mathcal{L}_0 \quad (3)$$

evaluated in the background (2). Then the black hole entropy is given by

$$S_{\text{BH}}(\vec{q}) = 2\pi (e^I q_I - f(\vec{u}, \vec{v}, \vec{e})) \quad (4)$$

at

$$\frac{\partial f}{\partial u_s} = 0, \quad \frac{\partial f}{\partial v} = 0, \quad \frac{\partial f}{\partial e^I} = q_I. \quad (5)$$

The first two sets of equations in (5) follow from equations of motion whereas the last equation follows from the definition of electric charge [5].

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<sup>1</sup>For a generic black hole often the string coupling constant at the horizon is fixed by the attractor mechanism and cannot be freely adjusted. We are implicitly assuming that we work with a black hole for which the string coupling constant at the horizon can be taken to be small either because it is not fixed or by adjusting some charges. Only in such cases the classical approximation makes sense. However once we arrive at a statistical interpretation of the classical Wald entropy using this approximation, we shall use the statistical entropy as the definition of Wald entropy in the full quantum theory.

Let us now make an analytic continuation  $t \rightarrow -i\tau$  to express (2) as a solution in Euclidean space-time. We get

$$ds^2 = v \left( r^2 d\tau^2 + \frac{dr^2}{r^2} \right), \quad \phi_s = u_s, \quad F_{r\tau}^{(I)} = -i e^I. \quad (6)$$

At the next step we introduce new coordinates  $(\eta, \theta)$  through the following series of transformations:

$$z = \tau + i r^{-1}, \quad w = (1 + iz)/(1 - iz), \quad \tanh(\eta/2)e^{i\theta} = w. \quad (7)$$

The complex coordinate  $z \equiv \tau + i r^{-1}$  describes the Euclidean  $AdS_2$  as an upper half plane, and the  $SL(2, \mathbf{R})$  isometries of  $AdS_2$  act on  $z$  as fractional linear transformations. In the  $w$  coordinate system the upper half plane is mapped into the interior of a unit disk. Finally  $(\tanh \frac{1}{2}\eta, \theta)$  are the usual polar coordinates on the unit disk in the  $w$ -plane. In the  $(\eta, \theta)$  coordinates the solution (6) appears as

$$ds^2 \equiv g_{\mu\nu}^E dy^\mu dy^\nu = v (d\eta^2 + \sinh^2 \eta d\theta^2) \quad \phi_s = u_s, \quad F_{\theta\eta}^{(I)} = i e^I \sinh \eta. \quad (8)$$

We now note that under the analytic continuation  $\theta \rightarrow i\tilde{t}$  and coordinate change  $\tilde{r} = \cosh \eta$ , (8) becomes

$$ds^2 = v [-(\tilde{r}^2 - 1)d\tilde{t}^2 + (\tilde{r}^2 - 1)^{-1}d\tilde{r}^2], \quad \phi_s = u_s, \quad F_{\tilde{r}\tilde{t}}^{(I)} = e^I. \quad (9)$$

This can be thought of as a black hole solution in  $AdS_2$  space with regular horizons at  $\tilde{r} = \pm 1$  [31–33]. This is in fact the solution that we get if we take a black hole close to extremality and examine its throat region [21, 22]. To see this we can take a Reissner-Nordstrom metric

$$ds^2 = -(1 - a/\rho)(1 - b/\rho)d\tau^2 + \frac{d\rho^2}{(1 - a/\rho)(1 - b/\rho)} + \rho^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (10)$$

and consider the limit  $\lambda \rightarrow 0$  at fixed  $(\tilde{r}, \tilde{t}, a)$  with

$$\rho = \lambda \tilde{r} + \frac{a + b}{2}, \quad b = a - 2\lambda, \quad \tau = a^2 \tilde{t} / \lambda. \quad (11)$$

In this limit the  $(\tilde{r}, \tilde{t})$  part of the metric (10) reduces to (9) with  $v = a^2$ . In contrast if we had taken the extremal limit  $a = b$  first and then taken the near horizon limit, we would have arrived at the metric given in (2)[7].

We now return to our analysis of the solution (8) which can be regarded as the Euclidean continuation of the black hole solution (9). The boundary of the unit disk in the  $w$  plane is at  $\eta = \infty$ , but we shall regulate the volume of  $AdS_2$  by putting an upper cut-off  $\eta_{\max}$  on  $\eta$ . Thus we take  $(\eta, \theta)$  to lie in the range

$$0 \leq \eta \leq \eta_{\max}, \quad 0 \leq \theta < 2\pi, \quad (\eta, \theta) \equiv (\eta, \theta + 2\pi). \quad (12)$$

Let us evaluate the classical action in this background. First note that the analytic continuation  $t \rightarrow -i\tau$  gives an extra factor of  $-i$  and replaces  $\sqrt{-\det g}$  by  $\sqrt{\det g}$ . Since  $\mathcal{L}_0$

is a scalar it remains unchanged under a coordinate transformation, i.e. the value of  $\mathcal{L}_0$  evaluated in the background (8) is the same as that evaluated in the background (2). According to (3), (4) this is given by  $(e^I q_I - (2\pi)^{-1} S_{\text{BH}}(\vec{q})) / v \cdot \sqrt{\det g} d^2x$  is also invariant under a general coordinate transformation. Thus on-shell the action is given by

$$\begin{aligned} \Gamma_0 &= -i \int d\eta d\theta \sqrt{\det g^E} \mathcal{L}_0 = -2\pi i v \int_0^{\eta_{\text{max}}} d\eta \sinh \eta \mathcal{L}_0 \\ &= -2\pi i (\cosh \eta_{\text{max}} - 1) f = i (\cosh \eta_{\text{max}} - 1) (S_{\text{BH}}(\vec{q}) - 2\pi e^I q_I). \end{aligned} \quad (13)$$

This is however not the complete contribution to  $\Gamma_0$ ; we can get additional contribution from the boundary terms at  $\eta = \eta_{\text{max}}$ . To determine the form of the boundary contribution, we make a change of coordinates

$$\tilde{\eta} = \eta_{\text{max}} - \eta, \quad \tilde{\theta} = \frac{1}{2} e^{\eta_{\text{max}}} \theta. \quad (14)$$

In these coordinates  $\tilde{\theta}$  labelling the coordinate along the boundary has period

$$\beta = \pi e^{\eta_{\text{max}}}. \quad (15)$$

Furthermore the metric and the gauge field strengths near the boundary take the form

$$\begin{aligned} ds^2 &= v \left[ d\tilde{\eta}^2 + \left( e^{-\tilde{\eta}} - e^{\tilde{\eta}-2\eta_{\text{max}}} \right)^2 d\tilde{\theta}^2 \right] = v \left[ d\tilde{\eta}^2 + e^{-2\tilde{\eta}} d\tilde{\theta}^2 \right] + \mathcal{O}(\beta^{-2}), \\ F_{\tilde{\eta}\tilde{\theta}}^{(I)} &= i e^I \left( e^{-\tilde{\eta}} - e^{\tilde{\eta}-2\eta_{\text{max}}} \right) = i e^I e^{-\tilde{\eta}} + \mathcal{O}(\beta^{-2}). \end{aligned} \quad (16)$$

Now the boundary term in the action is given by some local expression involving the various fields integrated along the boundary. Due to translation symmetry along  $\tilde{\theta}$ , the integration along the boundary gives a factor of  $\beta$  multiplying the integrand. On the other hand the form of the solution given in (16) shows that the integrand is given by a  $\beta$ -independent term plus a contribution of order  $\beta^{-2}$ . Thus up to correction terms of order  $\beta^{-1}$ , the boundary contribution must be proportional to the length  $\beta$  of the boundary circle.<sup>2</sup> Together with (13) this gives

$$\Gamma_0 = -i \left[ S_{\text{BH}}(\vec{q}) - 2\pi e^I q_I + \beta K(\vec{q}) + \mathcal{O}(\beta^{-1}) \right], \quad (17)$$

for some constant  $K(\vec{q})$ . Conventionally the terms linear in  $\beta$  are removed by adjusting the boundary terms [38], but we shall not need to worry about them. One point to note is that the  $\beta$  independent term in  $i\Gamma_0$  is precisely  $S_{\text{BH}}(\vec{q}) - 2\pi q_I e^I$  without any additional normalization factor; this will be important in what follows.

Let us compare this with the Euclidean action formalism [34]. According to this the action  $\Gamma_0$  is related to the energy  $E$ , entropy  $S_{\text{BH}}$ , charges  $q_I$  and the chemical potential  $\mu^I \equiv i \oint_{\tilde{\theta}} d\tilde{\theta} A_{\tilde{\theta}}^{(I)} \Big|_{\eta_{\text{max}}}$  via the relations:<sup>3</sup>

$$i\Gamma_0 = S_{\text{BH}} - \beta E + \mu^I q_I. \quad (18)$$

<sup>2</sup>This line of argument is similar to the one used in [38] in the context of  $AdS_3$  space.

<sup>3</sup>Note that our definition (5) of the electric charge is such that a point charge  $q_I$  will induce a term  $q_I \int A_{\mu}^{(I)} dx^{\mu}$  in the action.

Now for the classical background fields (8) we have

$$\begin{aligned} \oint_{\eta_{\max}} d\tilde{\theta} A_{\tilde{\theta}}^{(I)} &= - \int_{\eta \leq \eta_{\max}} d\eta d\theta F_{\theta\eta}^{(I)} \\ &= -2\pi i e^I (\cosh \eta_{\max} - 1) = -ie^I (\beta - 2\pi + \mathcal{O}(\beta^{-1})) . \end{aligned} \quad (19)$$

This gives

$$i\Gamma_0 = S_{\text{BH}}(\vec{q}) - 2\pi e^I q_I - \beta (E - e^I q_I) . \quad (20)$$

This agrees with (17) in the  $\beta \rightarrow \infty$  limit for the choice

$$E(\vec{q}) = -K(\vec{q}) + e^I q_I . \quad (21)$$

This is in accordance with the general result that for a regular black hole the Wald entropy and the one computed using Euclidean action formalism agree [1, 35]. Earlier exploration of the direct relation between the entropy function formalism and the Euclidean action formalism can be found in [39, 40].

We now return to our main goal, which is to give an interpretation of the entropy  $S_{\text{BH}}$  appearing in (17) in terms of an appropriate conformal quantum mechanics living at the boundary of  $AdS_2$ . For this we recall that  $e^{i\Gamma_0}$  is the classical partition function of the theory on  $AdS_2$ .<sup>4</sup> Since we are working in the approximation where the theory in the bulk is treated classically, one would expect this to be the partition function of the dual quantum mechanics living at the boundary  $\eta = \eta_{\max}$ . There is however one additional point we should remember. The usual rules of  $AdS/CFT$  correspondence [17, 18] tells us that for every gauge field  $A_\mu^{(I)}$  in the bulk we have a conserved charge  $Q_I$  in the boundary theory. Furthermore, in the presence of a non-zero  $A_\mu^{(I)}$  field at the boundary,  $e^{i\Gamma_0}$ , instead of calculating the partition function, is actually expected to calculate the expectation value of  $e^{iQ_I \oint d\tilde{\theta} A_{\tilde{\theta}}^{(I)}}$ . Now from (19) we have<sup>5</sup>

$$e^{iQ_I \oint d\tilde{\theta} A_{\tilde{\theta}}^{(I)}} = e^{Q_I e^I (\beta - 2\pi) + \mathcal{O}(\beta^{-1})} . \quad (22)$$

Thus if  $H$  denotes the Hamiltonian generating  $\tilde{\theta}$  translation<sup>6</sup> in the dual quantum mechan-

<sup>4</sup>Note that we have used the normalization and sign conventions appropriate for Lorentzian signature space-time. However the explicit  $-i$  factor in the expression (13) for  $\Gamma_0$  reflects that we are carrying out the path integral after analytic continuation to Euclidean signature space-time.

<sup>5</sup>Note that a change in  $\vec{e}$  not only induces a change in the boundary value of  $\{A_\mu^{(I)}\}$ , but also induces a change in the values of other fields via the attractor mechanism. The effect of all these other changes can be included in the Hamiltonian  $H$  of the boundary theory, thereby making  $H$  dependent on  $\vec{e}$ . Gauge fields are special, since besides the  $-i\beta e^I$  term in (19) which, being proportional to  $\beta$ , can be included as a correction to  $H$ , there is a  $\beta$  independent contribution  $2\pi i e^I$ . This cannot be regarded as a  $\beta$  independent correction to  $H$ .

<sup>6</sup>Since  $\tilde{\theta}$  translation induces a rotation about the origin in the  $w$ -plane,  $H$  can be identified with  $(L_{-1} + L_1)$  up to an additive constant and a proportionality factor.

ics living at  $\eta = \eta_{\max}$  then, according to *AdS/CFT* correspondence,<sup>7</sup>

$$e^{i\Gamma_0} = \text{Tr} \left( e^{-\beta H + (\beta - 2\pi) Q_I e^I + \mathcal{O}(\beta^{-1})} \right) = \text{Tr} \left( e^{-\beta H' - 2\pi Q_I e^I + \mathcal{O}(\beta^{-1})} \right), \quad H' \equiv H - e^I Q_I. \quad (23)$$

Since we are working in the classical limit  $Q_I$  can be replaced by the charge  $q_I$  carried by the black hole. Furthermore in the  $\beta \rightarrow \infty$  limit the right hand side of (23) gets its dominant contribution from the ground states of  $H'$ . If  $d(\vec{q})$  denotes the degeneracy of ground states of the  $CFT_1$  then in this limit eq. (23) takes the form

$$e^{i\Gamma_0} = e^{-\beta E'} d(\vec{q}) e^{-2\pi q_I e^I}. \quad (24)$$

Comparing this with (17), or equivalently (20), and noting that  $E' = E - e^I q_I$  as a consequence of (23), we see that the microscopic and the macroscopic results agree if we identify

$$e^{S_{\text{BH}}(\vec{q})} = d(\vec{q}). \quad (25)$$

Eq. (25) gives an interpretation of the Wald entropy  $S_{\text{BH}}(\vec{q})$  computed using the classical entropy function as the statistical entropy of the dual  $CFT_1$ .<sup>8</sup>

In the full quantum theory  $e^{i\Gamma_0}$  should be replaced by path integral over various fields of the theory in the  $AdS_2$  background. In order to properly define this path integral we need to fix the boundary condition on various fields at  $r = r_0$ . In  $AdS_{d+1}$  for general  $d$  the classical Maxwell equations for a gauge field near the boundary has two independent solutions. One of these represent the constant mode of the asymptotic gauge field and the other one measures the asymptotic electric field or equivalently the charge carried by the solution. Requiring the absence of singularity in the interior of  $AdS_{d+1}$  gives a relation between the two coefficients [17, 18]. Thus in defining the path integral over  $AdS_{d+1}$  we fix one of the coefficients and allow the other one to fluctuate. For  $d \geq 3$  the constant mode of the gauge field is dominant near the boundary; hence it is natural to fix this and allow the mode measuring the charge to be determined dynamically in the classical limit and to fluctuate in the full quantum theory. If we continue to define the partition function this way even for  $d = 1$  then the dual  $CFT_1$  will contain states of different charges. Let us denote by  $Z(\beta, \vec{e})$  the full quantum partition function with the boundary condition

$$-\frac{i}{2\pi} \oint d\theta (A_\theta^{(I)} - \partial_\eta A_\theta^{(I)}) = e^I, \quad (26)$$

as in (19). Note the subtraction term proportional to  $\partial_\eta A_\theta^{(I)}$ ; it removes the piece linear in  $\beta$  from  $\oint d\theta A_\theta^{(I)}$ . Equating  $Z(\beta, \vec{e})$  to the right hand side of (23) in the  $\beta \rightarrow \infty$  limit we

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<sup>7</sup>Conventionally one interprets the location of the boundary as providing an ultraviolet cut-off of the boundary theory. Here we shall find it more convenient to regard  $\beta$  as providing an infrared cut-off, keeping the ultraviolet cut-off fixed.

<sup>8</sup>Even though it appears that  $d(\vec{q})$  counts only the ground state degeneracies of the  $CFT_1$  we recall that we are working in units in which the ultraviolet cut-off has been taken to be of order 1. Thus all finite energy states in this unit are actually infinite energy states in the conventional unit in which the ultraviolet mass cut-off is taken to infinity. As a result ground states constitute the complete set of states of the  $CFT_1$ .

get<sup>9</sup>

$$Z(\beta, \vec{e}) = e^{-\beta E'} \sum_{\vec{q}} d(\vec{q}) e^{-2\pi q_I e^I}, \quad (27)$$

where  $d(\vec{q})$  is the degeneracy of ground states carrying charge  $\vec{q}$  in  $CFT_1$ . To see how this gives us back (25) in the semiclassical limit, we note that in this limit the left hand side is given by  $e^{i\Gamma_0}$ . Also in this limit  $d(\vec{q})$  is large, and the dominant contribution to the sum comes from the maximum of the summand as a function of  $\vec{q}$ . This gives

$$e^I = \frac{1}{2\pi} \frac{\partial \ln d(\vec{q})}{\partial q^I}, \quad i\Gamma_0 = -\beta E' + \ln d(\vec{q}) - 2\pi q_I e^I, \quad (28)$$

and

$$e^{i\Gamma_0} = e^{-\beta E'} d(\vec{q}) e^{-2\pi q_I e^I}. \quad (29)$$

Eq. (29) gives us back (24) and hence (25), whereas (28) recovers the relation between  $\vec{e}$  and  $\vec{q}$  given in (5).

We shall now argue however that  $Z(\beta, \vec{e})$  defined above is not the natural definition of the partition function in  $AdS_2$ . This is due to the fact that on  $AdS_2$  the mode that measures the electric field (and hence the charge) is the dominant one near the boundary. This can be seen for example in (19) where the term proportional to  $\beta$  measures the electric field and the constant term is determined in terms of the electric field by requiring the gauge fields to be non-singular at the origin. Hence fixing the constant mode of the gauge field at the boundary and letting the electric field (and hence the charge) to fluctuate amounts to integrating over non-normalizable modes. While such a definition of the partition function may work under certain circumstances, it is not guaranteed to work in general. Instead it is more natural to fix the asymptotic electric fields and let the constant mode of the gauge field fluctuate. Since fixing the asymptotic electric fields fixes the charges, the dual  $CFT_1$  will now only contain states of fixed charge, equal to that carried by the black hole. If we denote the  $AdS_2$  partition function defined this way by  $\widehat{Z}(\beta, \vec{q})$ , then (27) is replaced by

$$\widehat{Z}(\beta, \vec{q}) = e^{-\beta E'} d(\vec{q}) e^{-2\pi q_I \langle e^I \rangle}, \quad (30)$$

where  $e^{-2\pi q_I \langle e^I \rangle}$  is the expectation value of  $\exp[iq_I \oint d\theta (A_\theta^{(I)} - \partial_\eta A_\theta^{(I)})]$  at  $\eta = \eta_{\max}$ . Now for large  $r_0$  the subtraction term proportional to  $\partial_\eta A_\theta^{(I)}$  gives a fixed contribution of the form  $e^{C'\beta}$  for some constant  $C'$ . This leads to the simple formula:

$$d(\vec{q}) = \left\langle \exp \left[ -iq_I \oint d\theta A_\theta^{(I)} \right] \right\rangle_{AdS_2}^{finite}, \quad (31)$$

where  $\langle \rangle_{AdS_2}$  denotes the unnormalized path integral over various fields on  $AdS_2$  with fixed asymptotic values of the electric fields, and the superscript *finite* denotes that we need to pick the constant multiplicative factor of this expression removing all terms of the form

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<sup>9</sup>A similar equation was obtained in [41] using  $AdS_3/CFT_2$  correspondence. The analysis however depended heavily on supersymmetry, and the procedure used there to deal with the divergent terms is quite different from ours.



$e^{C_0\beta}$  for some constant  $C_0$ . In the classical limit  $\widehat{Z}$  reduces to  $e^{i\Gamma_0}$ , and  $\langle e^I \rangle = e^I$  where  $e^I$  and  $q^I$  are related by the attractor equation (5). Thus we again recover (25).

Eq. (31) can be used to compute  $d(\vec{q})$  in terms of the partition function of the appropriate string theory on  $AdS_2$ . It is then natural to define the entropy of the extremal black hole in the full quantum theory as  $\ln d(\vec{q})$ . This can then be compared with the statistical entropy computed using the microscopic description of the black hole.

Finally we would like to point out that for primitive charge vectors the quantum entropy defined this way refers strictly to the entropy associated with single centered black holes and will not capture the entropies of multi-centered black holes or any other configuration with the same total charge. This in particular will imply that this entropy will not suffer from wall crossing or the problems associated with entropy enigma [15].

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